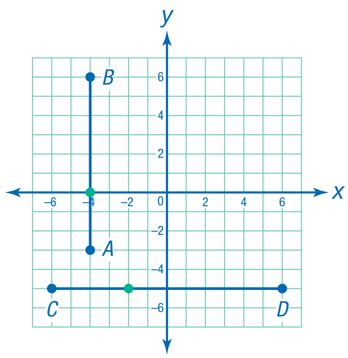
LESSON

21 Dividing Line Segments

UNDERSTAND The **midpoint** of a line segment divides, or **partitions**, the segment in half, producing two line segments of equal length, so the lengths have a ratio of 1:1. It is possible to find the point that divides a given line segment into two segments of any given ratio.

For a vertical or horizontal line segment, finding such a point is a straightforward process. Look at the coordinate plane.



Notice that all points on $\overline{AB}$ have the same x-coordinate, −4. So, the point $\frac{1}{3}$ of the way from A to B "rises" only $\frac{1}{3}$ of the way along the line. To find this point, add $\frac{1}{3}$ of the length of $\overline{AB}$ to the y-coordinate of A.

$$(-4, -3 + \tfrac{1}{3}AB) = (-4, -3 + \tfrac{1}{3} \cdot 9)$$
$$= (-4, -3 + 3) = (-4, 0)$$

The point (−4, 0) is $\frac{1}{3}$ of the way from A to B, and it partitions $\overline{AB}$ in a ratio 1:2.

A similar process can be used to find the point $\frac{1}{3}$ of the way from C to D. This point "runs" only $\frac{1}{3}$ of the length along the line from C to D.

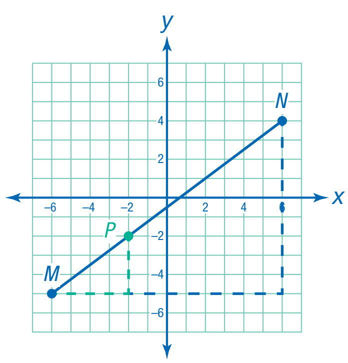
$$(-6 + \tfrac{1}{3}CD, -5) = (-6 + \tfrac{1}{3} \cdot 12, -5) = (-6 + 4, -5) = (-2, -5)$$

The point (−2, −5) is $\frac{1}{3}$ of the way from C to D, and it partitions $\overline{CD}$ in a ratio 1:2.

A diagonal line segment can also be partitioned by using a point. A point that is, for example, $\frac{1}{3}$ of the way from one endpoint to another "rises" $\frac{1}{3}$ of the way along the segment and also "runs" $\frac{1}{3}$ of the way along the segment.

Look at line segment $\overline{MN}$ on the coordinate plane below. To find the point P that is $\frac{1}{3}$ of the way from M to N, add $\frac{1}{3}$ of the "rise" to the y-coordinate of M and add $\frac{1}{3}$ of the "run" to its x-coordinate. Point M is located at (−6, −5), and N is located at (6, 4).

$$rise = 4 - (-5) = 9$$
$$run = 6 - (-6) = 12$$
$$P = (-6 + \tfrac{1}{3} \cdot 12, -5 + \tfrac{1}{3} \cdot 9) = (-6 + 4, -5 + 3)$$
$$= (-2, -2)$$



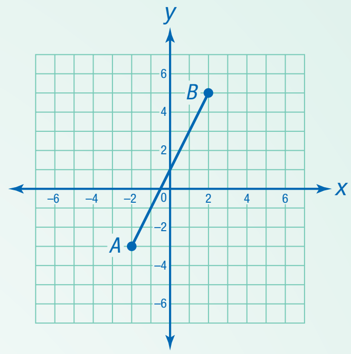
In general, for a line segment $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$, to find the point that partitions the segment in a ratio of m:n, or lies k of the way from A to B, use the following formula:

$$(x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1)) \qquad \text{where } k = \frac{m}{m+n}$$

Connect

The line segment $\overline{AB}$ is shown on the coordinate plane on the right.

Find the point Q that is $\frac{3}{4}$ the distance from A to B.
Then, plot and label Q on the coordinate plane.



1 Identify the endpoints of $\overline{AB}$.

The coordinates of the endpoints are $A(-2, -3)$ and $B(2, 5)$.

Since the problem states that Q is $\frac{3}{4}$ the distance from A to B, let $A = (x_1, y_1)$ and let $B = (x_2, y_2)$.

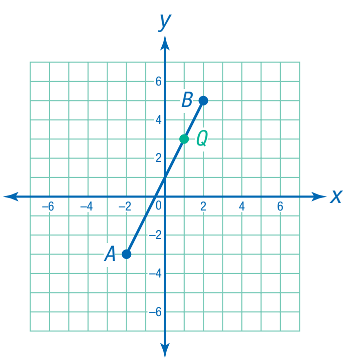
2 Use the formula to find point Q.

Let $k = \frac{3}{4}$, $(x_1, y_1) = (-2, -3)$, and $(x_2, y_2) = (2, 5)$.

$$Q = (x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$$
$$Q = (-2 + \tfrac{3}{4}[2 - (-2)], -3 + \tfrac{3}{4}[5 - (-3)])$$
$$Q = (-2 + \tfrac{3}{4}(4), -3 + \tfrac{3}{4}(8))$$
$$Q = (-2 + 3, -3 + 6)$$
$$Q = (1, 3)$$

▶ Point $Q(1, 3)$ is $\frac{3}{4}$ the distance from A to B.

3 Plot Q on the coordinate plane.

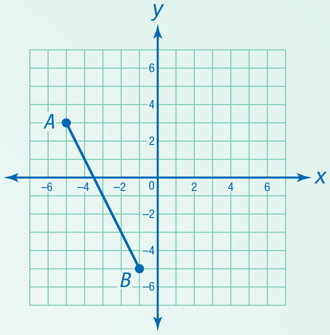


CHECK

Use the distance formula to find the lengths of $\overline{AQ}$ and $\overline{AB}$. Does $AQ = \left(\frac{3}{4}\right)AB$?

EXAMPLE A The line segment $\overline{BA}$ is shown on the coordinate plane on the right.

Find the point Q that partitions $\overline{BA}$ in a ratio of 1:3. Then, plot and label Q on the coordinate plane.



1 Identify the endpoints of $\overline{BA}$.

The coordinates of the endpoints are $A(-5, 3)$ and $B(-1, -5)$.

To partition $\overline{BA}$ in a ratio of 1:3, find the point that is $\frac{1}{1+3}$, or $\frac{1}{4}$, of the distance from B to A.

Let $B = (x_1, y_1)$ and let $A = (x_2, y_2)$.

2 Use the formula to find point Q.

Let $k = \frac{1}{4}$, $(x_1, y_1) = (-1, -5)$, and $(x_2, y_2) = (-5, 3)$.

$$Q = (x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$$

$$Q = (-1 + \tfrac{1}{4}[-5 - (-1)], -5 + \tfrac{1}{4}[3 - (-5)])$$

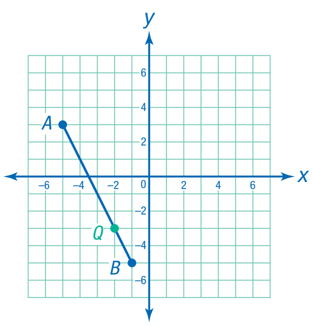
$$Q = (-1 + \tfrac{1}{4}(-4), -5 + \tfrac{1}{4}(8))$$

$$Q = (-1 + (-1), -5 + 2)$$

$$Q = (-2, -3)$$

▶ Point $Q(-2, -3)$ partitions $\overline{BA}$ in a ratio of 1:3.

3 Plot Q on the coordinate plane.

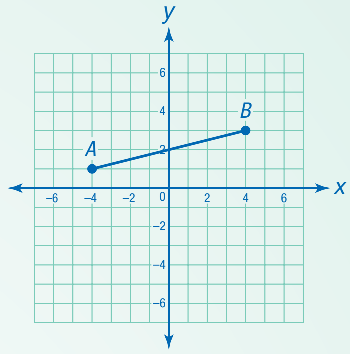


TRY

The endpoints of $\overline{CD}$ are $C(0, 3)$ and $D(12, 18)$. Find the point P that partitions $\overline{CD}$ in a ratio of 2:1.

EXAMPLE B The line segment $\overline{AB}$ is shown on the coordinate plane on the right.

Find the midpoint of $\overline{AB}$.



1 Identify the endpoints of $\overline{AB}$.

The coordinates of the endpoints are $A(-4, 1)$ and $B(4, 3)$.

The midpoint of $\overline{AB}$ is the point that is $\frac{1}{2}$ of the distance from A to B.

Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$.

2 Use the formula to find the midpoint.

Let Q be the midpoint of $\overline{AB}$.

Let $k = \frac{1}{2}$, $(x_1, y_1) = (-4, 1)$, and $(x_2, y_2) = (4, 3)$.

$$Q = (x_1 + k(x_2 - x_1), y_1 + k(y_2 - y_1))$$

$$Q = (-4 + \tfrac{1}{2}[4 - (-4)], 1 + \tfrac{1}{2}(3 - 1))$$

$$Q = (-4 + \tfrac{1}{2}(8), 1 + \tfrac{1}{2}(2))$$

$$Q = (-4 + 4, 1 + 1)$$

$$Q = (0, 2)$$

▶ Point $(0, 2)$ is the midpoint of $\overline{AB}$.

DISCUSS

The formula for finding the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

How does this formula relate to the formula that you have been using?

Practice

Find the coordinates of point *Q*.

1. The line segment $\overline{AB}$ is shown on the coordinate plane on the right.

Find the point *Q* that is $\frac{1}{5}$ the distance from *A* to *B*.

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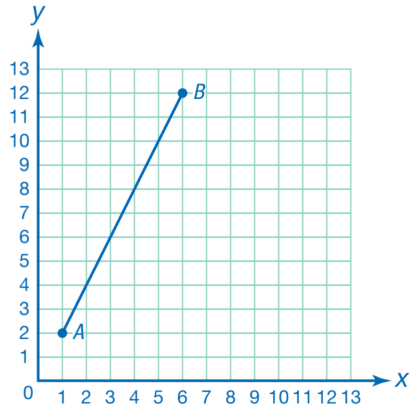

2. The line segment $\overline{CD}$ is shown on the coordinate plane on the right.

Find the point *Q* that is $\frac{2}{3}$ the distance from *C* to *D*.

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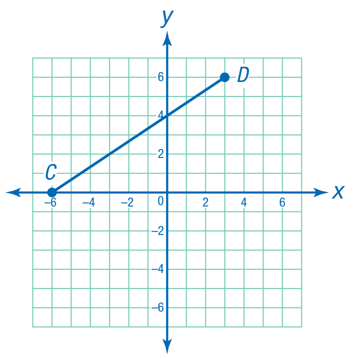

3. The line segment $\overline{GF}$ is shown on the coordinate plane on the right.

Find the point *Q* that partitions $\overline{GF}$ in a ratio of 1:3.

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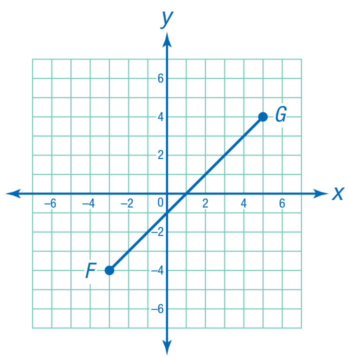

4. The line segment $\overline{JK}$ is shown on the coordinate plane on the right.

Find the point *Q* that partitions $\overline{JK}$ in a ratio of 3:2.

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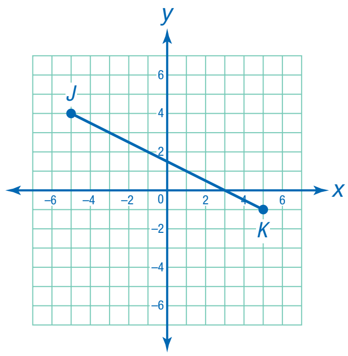


Use the information below for questions 5 and 6. Find the point described.

The endpoints of line segment $\overline{XY}$ are $X(-6, 2)$ and $Y(6, -10)$.

5. Find the point P that is $\frac{1}{3}$ the distance from X to Y. ______________

6. Find the point Q that partitions $\overline{YX}$ in a ratio of 3:1. ______________

Solve.

7. Point A is located at (1, 4). Point P at (3, 5) is $\frac{1}{3}$ the distance from A to point B.

What are the coordinates of point B? ______________

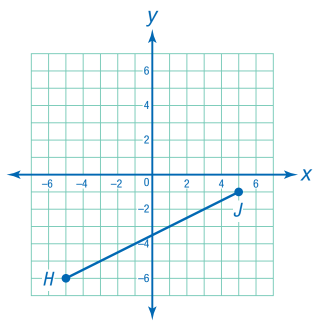
8. Point C is located at the origin. Point Q at $(-1, -2)$ partitions $\overline{CD}$ in a ratio of 1:6.

What are the coordinates of point D? ______________

9. The line segment $\overline{HJ}$ is shown on the coordinate plane to the right.

Find the point Q that is $\frac{4}{5}$ the distance from J to H. ______________

Plot point Q on $\overline{HJ}$.



Fill in the blank.

10. REASON If point P is $\frac{3}{7}$ the distance from A to B, then it is _____ the distance from B to A.

Plot points *L* and *P* as described.

11. SHOW Point K is shown on the coordinate plane on the right.

Plot a point L so that it is 15 units from point K and so that $\overline{KL}$ is not vertical or horizontal. (Hint: Find a Pythagorean triple where the largest number is 15.) Then, add point P that is $\frac{1}{3}$ the distance from K to L.